MINIMIZATION OF SPUR GEAR DYNAMIC LOADING THROUGH THE GENERALIZED THEORY OF GEARING

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SYNOPSIS

An analysis was conducted to investigate the effects of tooth profile modification on the load response of low contact ratio straight teeth cylindrical gears. The generalized theory of gearing is used to determine gear dimensioning. Tooth loading results from the static load analysis whereas an optimized profile modification is found to extinguish the abrupt load changes and minimize the dynamic loading.

1. INTRODUCTION

In the design of power transmitting gears the reduction of tooth dynamic loading is one of the major facets. Research on gear noise and vibration, [1] and [2], has revealed that this noise is basically generated through the gearbox excitation by the dynamic loading. Vibration is transmitted by the shafts and bearings to the noise radiating external surfaces. The tooth roots suffer cyclic bending stresses (which lead to fatigue failure) and cyclic subsurface stresses (which lead to surface failure) by pitting or scoring due to dynamic tooth loading. Minimization of gear dynamic loads decreases gear noise, increases efficiency, impoves pitting fatigue life and prevents the tooth's fracture, [3] and [4]. The present paper examines the basic dimensioning of the gears through the generalized theory of gearing and static analysis, which is followed by the necessary calculations of tooth stiffness and modifications of tooth flank, in such a way that abrupt changes in load distribution have be. minimized for varying gear parameters. Compliance of th. teeth of two spur gears, as the teeth work together along the path of contact, from the very beginning to the end of their contact, has been examined by [3], [5] and [6]. However, these or any other relative publications give results only for the unusual case of a gear pair with gear ratio equal to one (two identical gears in mesh). No result or detail of calculation of compliance is given for the most usual situation of two unequal gears in mesh namely, a pinion and a gear). This work presents a way of calculation of teeth compliance as a function of gear ratio, of the number of teeth of pinion, of the addendum, of pressure angle etc. Tooth load distribution is evaluated for the above given parameters. It is proved that gears in mesh with contact ratio between 1 and 2 may be, under certain circumstances, without dynamic loading due to abrupt loads. In power transmission gear - systems a lot of research is directed in increasing power-to-weight ratio and reliability. It may .e done through the use of the available materials . by developing new ones or by improving manufacturing methods. Alternatively, more efficient tooth geometries may be developed, other than the generally used standard involute or Wildhaber - Novicov tooth flanks, [7] to [9]. The generalized theory of gearing, [10] and [11], allows the calculation of gear geometry through the use of any combination of permissible tooth flank profiles. In connection to this direction (more efficient geometries) the present work examines the design and evaluation of a pair of spur gears through the working and existence conditions of such a pair, [12].

2. TOOTH PAIR COMPLIANCE

In a gear pair, cooperating teeth push each other in the direction of the line of action. In this way, displacements of the teeth happen, along the line of action, that on the cooperating teeth create situations similar to those resulting from errors of construction on the tooth flank. The elastic deflection characteristics of an individual gear tooth can best be described in terms of a gear tooth compliance coefficient. This coefficient is defined as the relative deflection, measured normal to the tooth profile and along the line of action, per unit force applied per unit gear face width. Compliance coefficient is given by the relationship

$$\mathbf{C = \delta Eb/W}$$

where **C** = nondimensional tooth compliance, **b** = face width of the gear, **E** = modulus of elasticity, **d** = normal tooth deflection along the line of action, **W** = normal tooth load along the line of action. This load **W**, acting on a tooth as in Fig.1, may produce on the tooth three different types of deformation: 1) Displacement of the tooth axis of symmetry with the root of tooth being unflexible. It is a bending deflection that creates the bending compliance C_B. 2) Local surface deformation due to Hertzian compression that creates the Hertzian compliance C_H. 3) Deflection of the tooth's root due to flexibility of the tooth's base, which is due to foundation flexibility and creates the foundation compliance C_F.

3. GENERALIZED THEORY OF GEARING

The generalized theory of gearing examines the exact flank geometry of the teeth of spur gears as it is generated by the cutting process of a rack, in such a way that from the known rack geometry, pinion's flank is found to which, through the conjugate action of gearing, the pertinent geometry of the external or internal mating gear is evaluated. The cutting process of a rack, of a known geometry **y = F(x)**, may be simulated by the generalized theory of gearing, [10], to calculate: **i)** pinion's tooth flank **ii)** the external or internal flank of the mating with the pinion gear and **iii)** the path of contact of these mating gears. The present simulation, shown in Fig. 2, takes the rack profile **y = F(x)** in the coordinate system (x,y) whose origin is at the pitch point **P(0,0)**. The **x**-axis coincides with the rack pitch line and the positive **x**-axis shows the direction of the rack's motion. In this case the pinion 1 has a clockwise rotation. Point **A(x,y)** of the rack profile corresponds to the point $\mathbf{B(x_{pc}, y_{pc})}$ of the path of contact and to the point $\mathbf{C(x_1, y_1)}$ of the pinion tooth profile. It is obvious that point **A** of the rack profile, mc. a distance **K**, and point **C** of the pinion tooth profile, rotated through the angle (θ), will coincide at point **B** of the path of contact. The law of gearing states that the common normal to the mating profiles at the point of contact must pass through the pitch point **P(0,0)**. Thus,

$$\tan\alpha = -dx/dy = y_{pc}/x_{pc}$$

From this and with the no-slip condition of the pitch lines one may receive

$$K = -(y \cdot dy/dx + x) = (\theta) \cdot R_{01}$$

The corresponding point $C(x_1, y_1)$ of the pinion profile is found by the relations:

$$x_1 = (x + K) \cdot \cos(\theta) - (y + R_{01}) \cdot \sin(\theta)$$
$$y_1 = (x + K) \cdot \sin(\theta) + (y + R_{01}) \cdot \cos(\theta) - R_{01}$$

where, for the module m of the gearing and the number of teeth Z_1 of pinion, the pinion pitch radius is given by $R_{01} = m \cdot Z_1/2$.

The profile of the mating gear depends on the kind of this gear, whose teeth **Z** may be internal or external (i.e. Z_{in} or Z_{ex}). When the teeth of the mating gear are internal, the center of the gear lies on the same side of y-axis as the center of pinion and this gear rotates to the same direction (here clockwise). If the pitch radius of the internal gear is $R_{o,in} = m \cdot Z_{in}/2$ then for its angle of rotation (θ_{in}) it is true that

$$K = (\theta) \cdot R_{01} = (\theta_{in}) \cdot R_{o,in}$$

The corresponding point $C_{in}(x_{in}, y_{in})$ of the mating internal gear profile is given by

$$x_{in} = (x + K) \cdot \cos(\theta_{in}) - (y + R_{0,in}) \cdot \sin(\theta_{in})$$
$$y_{in} = (x + K) \cdot \sin(\theta_{in}) + (y + R_{0,in}) \cdot \cos(\theta_{in}) - R_{0,in}$$

When the teeth of the mating gear are external, the center of the gear lies on the other side of y-axis and this gear rotates in the opposite direction (here counterclockwise). If the pitch radius of the external gear is $R_{o,ex} = m \cdot Z_{ex}/2$ then for its angle of rotation (θ_{ex}) it is true that

$$K = (\theta) \cdot R_{01} = -(\theta_{ex}) \cdot R_{o,ex}$$

The corresponding point $C_{ex}(x_{ex}, y_{ex})$ of the mating external gear profile is given by

$$x_{ex} = (x + K) \cdot \cos(\theta_{ex}) - (y + R_{0,ex}) \cdot \sin(\theta_{ex})$$
$$y_{ex} = (x + K) \cdot \sin(\theta_{ex}) + (y + R_{0,ex}) \cdot \cos(\theta_{ex}) - R_{0,ex}$$

The rack data and its geometric parameters

The rack modelling is achieved by using the following coefficiends the addendum coefficient C_k, the dedendum coefficient C_f, the coefficient of profile shifting C_m, the coefficient of rack's tooth tip radius C_c, the coefficient of tooth thickness at pitch radius C_s, the module **m** and the pressure angle α_o. The rack profile **y = F(x)** and the above calculated gear flanks and path of contact are subject to the following notations: Addendum $h_k = mC_k$, dedendum $h_f = mC_f$ pitch circle radius $R_o = Zm/2$, rolling circle radius $R_b = R_o + mC_m$, base circle radius $R_g = R_o\cos(\alpha_o)$ root circle radius $R_f = R_b - mC_f$, outside circle radius $R_k = R_b + mC_k$, circular pitch $t_o = \pi m$, tooth thickness at pitch circle $S_o = t_oC_s$, tooth thickness at pitch circle after shifting $S_{ov} = S_o + 2mC_m\tan(\alpha_o)$, rack's tooth tip radius $R_c = mC_c$.

4. CALCULATION OF COMPLIANCES

Bending and foundation compliances have to be calculated for a tooth of each gear, whereas Hertzian compliance is evaluated for the junction of both teeth as they touch each other.

4.1 Bending compliance, C_B

For the calculation of bending deflection, a gear tooth is taken as an elastic beam based on rigid foundation, Fig.1. Load **W**, acting in the direction of the normal at the tooth flank, is analyzed into forces **Q** and **N**. For the calculation of C_B, bending deflection δ is needed, which is found by equating the external work done by the load **W** to the internal energy, as follows:

$$W \cdot \delta = \int_0^Y [(M^2/E{\cdot}I) + (K{\cdot}Q^2/G{\cdot}A) + (N^2/E{\cdot}A)]\,dy$$

where: $A = b \cdot t$ = tooth cross sectional area, $K = (12 + 11\nu)/(10 + 10\nu)$, $Q = W \cdot \cos\varphi$ = shear load component, $N = W \cdot \sin\varphi$ = radial load component, $M = -(Y - y) \cdot Q$ = bending moment, $I = b \cdot t^3/12$ = area moment of inertia, E = modulus of elasticity, $G = E/2(1 + \nu)$ = shear modulus, ν = Poisson's ratio.

By using the definition of C_B and putting it into the above equation we may write

$$C_B = Eb\delta/W = (\cos\phi)^2 \{12 \cdot INT_2 + [2.4 + 2.2\nu + (\tan\phi)^2]\,INT_1\}$$

where the integrals

$$INT_1 = \int_0^Y (1/t)\,dy \quad \text{and} \quad INT_2 = \int_0^Y [(y - Y)^2/t^3]\,dy$$

may be calculated numerically. So, the bending compliances C_{Bp} for pinion **p** and C_{Bg} for the gear **g** are found. The angle φ is calculated from the position of load **W**. Fillet and involute relations from the generalized theory of gearing help to determine **t** at any position **y**.

4.2 Hertzian compliance C_H

For the tooth compression, the generally accepted for gearing contact theory of Hertz says that at any point **E** of two cooperating teeth their flanks resemble cylinders with radii ρ_1 and ρ_2, as Fig. 3 where **K** and **Ē** are the common points of the path of contact with the base circles of the gears. Each tooth is compressed by force **W** and it is assumed that the compression effect of this load extends from point **E** of contact up to the tooth's centerline. Beyond $h_1 = EK_1$ and $h_2 = EK_2$ the load is transmitted as a ctransverse shear force to the rest of the gear body. The deflection in this case is calculated, as in [13], by the formula:

$$\delta_H = \{\ln(4h_1h_2/c^2) - \nu/(1-\nu)\}\,(1 - \nu^2)\,2W/\pi bE$$

where

$$c^2 = 8W\rho_1\rho_2(1 - \nu^2)/\{\pi bE(\rho_1 + \rho_2)\}$$
$$\nu^2 = 1 - \{(1 - \nu_p^2)/E_p + (1 - \nu_g^2)/E_g\}\,E_pE_g/(E_p + E_g)$$
$$E = 2E_pE_g/(E_p + E_g)$$

where E_p, ν_p and E_g, ν_g are the modulus of elasticity and Poisson's ratio for pinion **p** and gear **g** respectively. For gears with external teeth, the maximum developed Hertzian pressure p_{max} may be calculated, [12], as follows:

$$p_{max}^2 = WE(\rho_1 + \rho_2)/2\pi b(1 - \nu^2)\rho_1\rho_2 \leq p_{al}^2$$

where p_{al} is the allowable pressure. Therefore,

$$c = 4\rho_1\rho_2(1 - \nu^2)p_{max}/[E(\rho_1 + \rho_2)]$$

Hertzian compliance is found by $C_H = Eb\delta_H/W$ and therefore,

$$C_H = \{\ln(4h_1h_2/c^2 - \nu/(1 - \nu)\}\,2(1 - \nu^2)/\pi$$

4.3 Foundation compliance C_F

The deflection of a tooth at the position a tooth at the position of its load due to the deflection and rotation of the tooth's base may be found by treating the tooth as a rigid beam built into an elastic foundation and equating the deformation work of the external load W with the internal stress energy of the foundation. It is found that the foundation compliance is given, [14], by the following relation:

$$C_F = (A_1 + A_2 + A_3)(1 - \nu^2)(\cos\phi)^2$$

where ν here is ν_p or ν_g

$$A_1 = (50/3\pi)(Y/t_F)^2$$
$$A_2 = [2(1 - 2\nu)/(1 - \nu)](Y/t_F)$$
$$A_3 = (4.82/\pi)\{1 + [(\tan\phi)^2/2.4(1 + \nu)]\}$$

It depends on φ, Y and t_F, Fig.1. The foundation width t_F is calculated through x_o, which is half of the tooth's width in the position where the fillet meets the root diameter of the gear. Fillet's coordinates are referred to system **XOY**, where **O** is the gear's center, axis **OY** is the axis of symmetry of a tooth and **OX** is perpendicular to **OY** to the right. It is true that $t_F = 2|x_o|$. Through the generalized theory of gearing, [10], we may have

$$x_o = R_c \cdot \cos w - (R_o + R_c - h_f) \cdot \sin w$$
$$y_o = R_c \cdot \sin w + (R_o + R_c - h_f) \cdot \cos w$$
$$w = \{C_s + 2(C_f - C_c)\tan\alpha_o + [2C_c/\cos\alpha_o]\}/Z$$

Fig.4 shows all kinds of compliance for a pair of standard (i.e. addendum coefficient equal to unity) gears in mesh, for which $Z_p = 22$ and $Z_g = 35$ and the contact ratio is $\varepsilon = 1{,}63$.

The total deflection of a pair of teeth, that is due to the total compliance, is the sum of the individual deflections of each tooth, that is

$$\delta_{tot} = \delta_{Bp} + \delta_{Fp} + \delta_{Bg} + \delta_{Fg} + \delta_H = CW/bE$$

By using the definition of compliances, this equation becomes as follows and gives results as those of Fig.4:

$$C = C_H + 2\{E_g \cdot (C_{Bp} + C_{Fp}) + E_p \cdot (C_{Bg} + C_{Fg})\}/(E_p + E_g)$$

5. LOAD DISTRIBUTION

In standard spur gears the load of a gear is shared by one or two teeth. Referring to Fig. 5, each tooth pair starts engagement at A, $(x = 0)$, and stops at B, $(x = \varepsilon)$. During (AA') and (B'B) there are two pairs of teeth in cooperation, each bearing its part of the total transmitted load **P** of the gear, whereas during (A'B') there is only one pair of teeth in contact. Let 1 refer to the first pair of teeth and 2 to the second pair of teeth in contact and

W_1, W_2 = normal load on pair 1 and 2 respectively;
C_1, C_2 = compliance of the pair 1 and 2 respectively;
e_1, e_2 = relative manufacturing and / or operational errors on the gear flank profile;
L_1, L_2 = profile modifications (positive or zero).

In meshing gears, when two pairs of teeth are in contact, the transmission errors at the points of contact have to be equal in order to avoid interference. So,

$$\delta_1 + e_1 + L_1 = \delta_2 + e_2 + L_2 \quad \text{or}$$
$$W_1 \cdot C_1 / b \cdot E + e_1 + L_1 = W_2 \cdot C_2 / b \cdot E + e_2 + L_2$$

But $W_1 + W_2 = W$. These two equations give:

$$W_1 = \{W \cdot C_2 + b \cdot E \cdot (e_2 + L_2 - e_1 - L_1)\}/(C_1 + C_2)$$
$$W_2 = \{W \cdot C_1 - b \cdot E \cdot (e_2 + L_2 - e_1 - L_1)\}/(C_1 + C_2)$$

for the cases of two pairs of teeth in contact, whereas for the case of one pair of teeth in contact $W_1 = W$.

5.1 Error free operation

When no errors exist on the gear flanks and no profile modifications have been made, we have $e_1 = L_1 = e_2 = L_2 = 0$. Then, using the notation of Fig.6 the previous relations may be used to find the load distribution as: $W_{b+} = W_{c-} = W$ and

$$W/W_a = 1 + (C_a / C_c), \qquad W/W_{b-} = 1 + (C_b / C_d)$$
$$W/W_{c+} = 1 + (C_c / C_a), \qquad W/W_d = 1 + (C_d / C_b)$$

These loads constitute abrupt changes in the tooth load and they cause inertial phenomena which have to be eliminated.

5.2 Operation with profile modification

Profile modification of the teeth is an intentional change of the involute teeth flank for the elimination of the abrupt changes in the gear load distribution. Initially, the load is taken to be equal to zero for the end points **A** and **B** of the path of contact. Supposing that the teeth are error-free and that modification will be done only at the areas of 2 - pair contact, **ab** and **cd**, Fig. 7, the area of 1 - pair contact is kept with involute profiles. Let the path of contact have the modification of the form:

$$\lambda/\lambda_a = [1 - (x/x_a)]^z \quad \text{for} \quad 0 \le x \le x_a$$
$$\lambda = 0 \quad \text{for} \quad x_a < x < \varepsilon - x_d$$
$$\lambda/\lambda_d = [1 - (\varepsilon - x)/x_d]^z \quad \text{for} \quad \varepsilon - x_d \le x \le \varepsilon$$

where: $z \ge 1$, ε = the contact ratio of the gear pair, x_a, x_d = length of modification on the path of contact, Fig. 7, and λ_a, λ_d = the modifications of the path of contact at end-points **a** and **d** respectively.

These modifications change the load distribution along the path of contact. If the load at the points **a** and **d** is taken to be zero, and having in mind the previous assumptions, we may write: $e_a = e_b = e_c = e_d = 0$, $\lambda_b = \lambda_c = 0$ and $W_a = W_d = 0$. From this we have: $W_1 = W_a = 0$ and $W_2 = W_d = 0$. Then,

$$L_a = W \cdot C_c / b \cdot E \quad \text{and} \quad L_d = W \cdot C_b / b \cdot E$$

Fig. 8 shows the results for several values of z.

These modifications have an impact on the tooth profile. The change of the tooth profile geometry is calculated through the generalized theory of gearing in such a way that the corresponding tooth profile modifications L_1, L_2 are found.

6. DISCUSSION AND CONCLUSION

Modification of the path of contact changes the load distribution of meshing gears by changing the goemetry of the gear flanks, which may be determined through the use of generalized theory of gearing. In this way, the effect of the profile modification on the performance of spur gears has been exactly found. It is observed that, for the the larger gear, the minimum value of bending compliance is smaller than the minimum value of the same compliance of pinion. This is in agreement with the experimental results of [15]. Moreover, total compliance decreases as the number of teeth of the larger gear and/or the number of teeth of pinion increases, which is explained by the fact that the tooth root thickness increases, relative to tooth thickness in the pitch diameter, as the number of teeth increases. It is also verified that compliance is independent of the diametral pitch, as was experimentally found by [15]. As far as load distribution is concerned, the existing load jumps, that produce inertial phenomena, during the gear operation, may be eliminated. The calculated here profile modifications have made it possible to minimize these phenomena. It is proved that gears with special toothing and S shaped path of contact with linear or parabolic tooth profile modification have considerable advantages compared to those with involute teeth. These advantages are related to the higher relative velocities and better meshing of teeth that are met at the ends of the path of contact.

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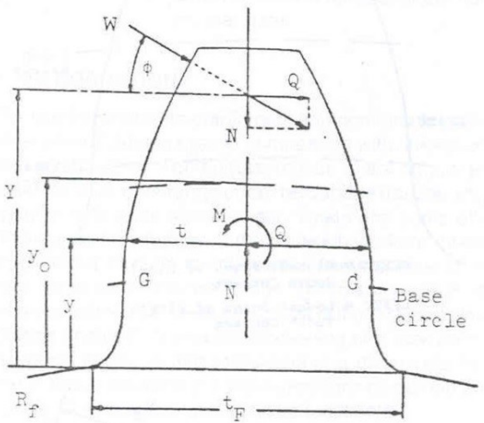

Fig. 1 A loaded gear tooth

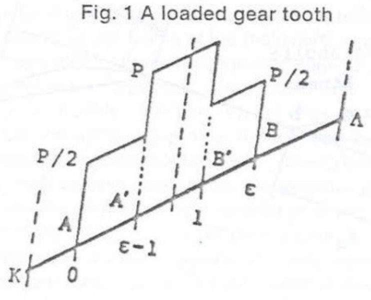

Fig. 5 1-pair and 2-pair tooth loading

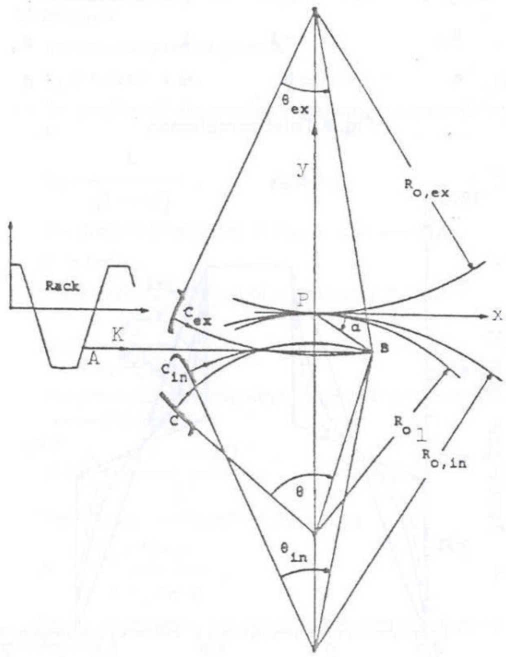

Fig. 2 Generating process of gear flanks

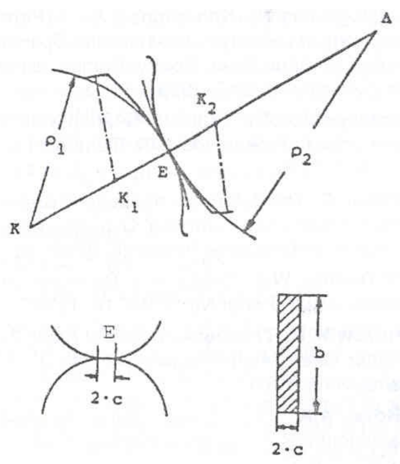

Fig. 3 Equivalent cylinders

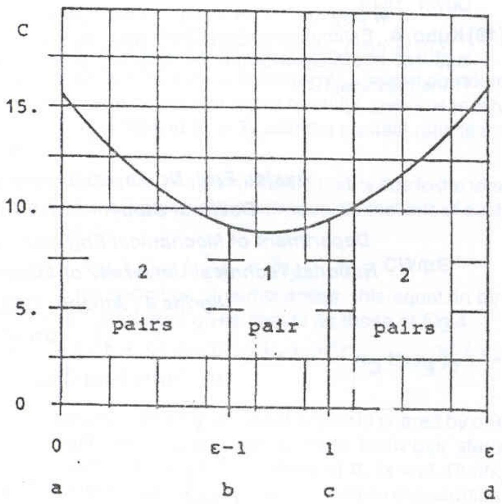

Fig. 6 Total compliance

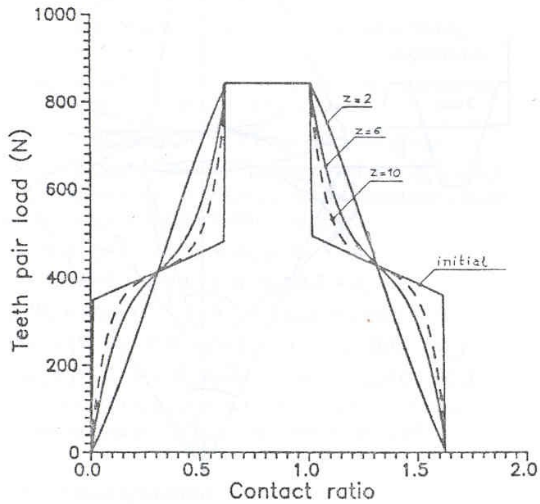

Fig. 8 Effect of profile modification on load distribution

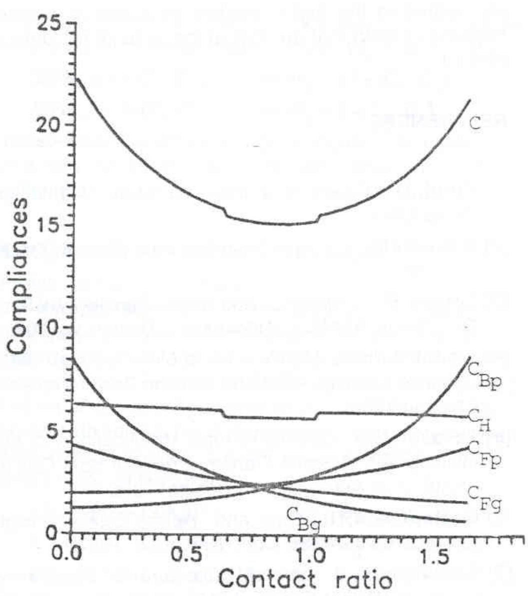

Fig. 4 Compliances

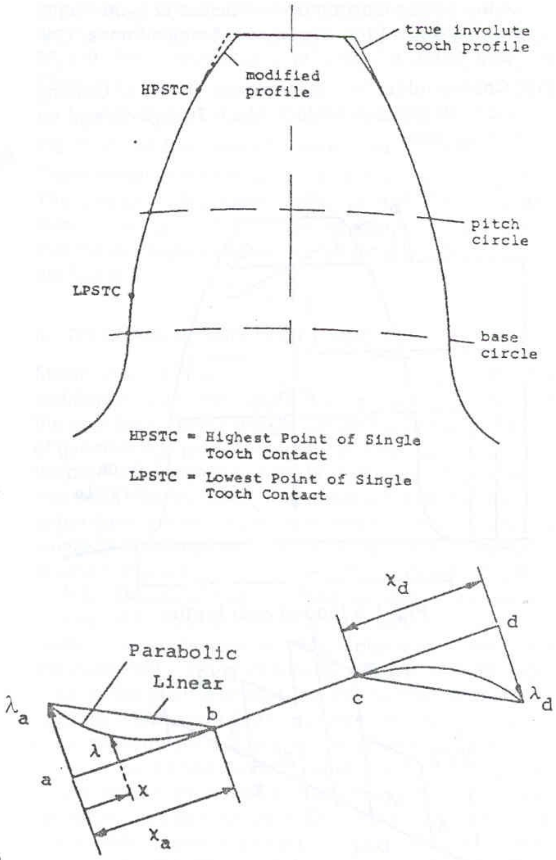

Fig. 7 Modified path of contact and gear tooth modification